General Physics 1

Physics 201 *Spring 2014*

Accurate but Honest Answers with "Sig Figs"

Measurements of all physical quantities are limited in precision. The digits that are known to be correct are called "significant figures," or "sig figs". It is important to keep track of the sig figs in a calculation, and a calculator cannot do all the work.

There are two competing goals:

1. To compute as exactly as possible.
2. To be truthful about the limitations of your input data

For example, suppose that you want to measure the length of a particular table, but since it is slightly longer than one meter you do it in two steps. You first measure with a meter stick the distance from one end of the table to a particular mark on the table near the other end. That distance turns out to be 0.95 meters. Then you use a more precise ruler to determine that the distance from the mark to the other end of the table is 0.1153 meters. You would then think that the length of the table is

```
  0.95
+ 0.1153
---------
= 1.0653
```

But the first measurement was only known to 2 places past the decimal point. You don't really know that the digits past that are zero – they could be anything. So only the first two digits past the decimal are "significant", and the final result can only be known to that precision. To get the closest answer to the sum above we **round** the answer to two places past the decimal. So the best value we can get for the length of the table is 1.07 meters.

A useful way to think of this is that the first measurement was really "`0.95????`" while the second measurement was "`0.1153???`", where the "`?`" indicates a digit which we don't know. When you add the digits in each column, you can treat the "`?`" as a zero, since you don't know for sure what that digit should really be. Then any colum that has a "`?`" in it is considered to be "tainted" and should not be trusted. Thus:

```
  0.95???
+ 0.1153?
---------
= 1.0653?
= 1.07???
= 1.07
```

Notice that we didn't just truncate the result, we rounded up. After all, the 0.1153 is closer to 0.12 than it is to 0.11, so the final result should be closer to 1.07 than 1.06.

You can easily convince yourself that the same considerations apply to subtraction. Any column containing a "?" is tainted, and so we round to the last untainted column of digits. Thus we are lead to the following rule for addition and subtraction:

Rule for Addition and Subtraction:

When adding or subtracting, perform the operation as usual, but restrict your result by rounding to the smallest number of *digits past the decimal* in any term.

Next consider multiplication. Following the previous example, suppose that we want to know the area of a particular rectangular region on the table-top. With the meter stick we measure the length of one side of the rectangle to be 0.47 meters, and with the more precise ruler we measure the other side to be 0.315 meters. The area is then

```
   0.47
 x 0.315
-----------
 = 0.14899
```

The first number only has two digits, while the second number has three, and the result has 5 digits. How many digits in the result are actually meaningful?

Again, think of the missing digits is as "?". You can work out the multiplication of these two numbers the 'long' way (digit by digit), and keep track of which columns get tainted by "?"s. In the end you'll find that the "?" in the shortest number will taint the result after the same number of digits. That is, when you multiply by "0.65??" there are two good digits in the result, and then the third one will be tainted by the "?". The result will therefore look like:

```
   0.47??
 x 0.315?
-------------
 = 0.14899
 = 0.15???
 = 0.15
```

Again, notice that when we restricted the answer to the two good digits we **round**, not just truncate.

Similar consideration apply to division. If you work out your result by long division and keep track of which column is tainted by a "?", the result will be limited by the number with the fewest total digits. This leads us to the following rule for multiplication and division:

Rule for Multiplication and Division

> When multiplying or dividing, perform the operation as usual, but restrict your result by rounding to the smallest number of *digits from the beginning of the number* in any operand.

Now consider what happens when you perform more complicated calculations. You might subtract two numbers, then multiply by another, then add that to another number. If you round the result of each operation to just a few digits you could be throwing away more valuable information than you should. If you round at each step you could also find that you get slightly different results depending on the order in which you perform the operations, even when you should get the same results. In order to get an answer which is as accurate as possible you should use the most accurate values in intermediate calculations, which means that it is okay to keep extra digits beyond the "significant" figures. This leads to the rule for repeated calculations:

Rule for Repeated Calculations

> Use as many digits as possible in *intermediate* calculations, but round to the appropriate number of "sig figs" for the final answer.

In other words, you should only apply the rules above to round the result to the "significant" digits when you report a result, but not for values used "internally" in a calculation.

One final point is important: these rules are only approximations – they are "rules of thumb", that work well for getting a good rough estimate of the precision of a numerical result. They are not a replacement for a careful analysis of the propagation of errors through a calculation, but they are usually enough for most practical purposes.

Summary of The Rules for Sig Figs

1. When adding or subtracting, perform the operation as usual, but restrict your result by rounding to the smallest number of *digits past the decimal* in any operand.
2. When multiplying or dividing, perform the operation as usual, but restrict your result by rounding to the smallest number of *digits from the beginning of the number* in any operand.
3. Use as many digits as possible in *intermediate* calculations, but round to the appropriate number of "sig figs" for the final answer.